

Application of Type-2 Fuzzy Logic – A Review

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Abstract—In this paper, a comprehensive survey on various applications of Type-2 Fuzzy Logic has been carried out. GPU (Graphics Processing Units) is used for speed up purpose, otherwise commonly used for graphics applications. However, recent trends show use of GPU in various other general computational applications to run them parallel to reduce overall execution time. In this paper, it is discussed that how GPU and fuzzy logic can together be helpful in solving problems of different domains for faster responses.

Keywords—GPU, CUDA, FLS, Type II Fuzzy Logic

I. INTRODUCTION

Latest advancements in parallel computing exploit GPGPUs that have multi-core architecture which supports parallel computations especially required for graphical processing. They devote more transistors for arithmetic and logical operations as compared to data caching and flow control compared to a CPU. Due to processing demand GPUs have advanced rapidly. And also beating the CPUs in terms of number of cores and hence, their computational power. As NVIDIA has launched CUDA software development kit in 2007, the use of GPU's computational powers for general purpose computing has become easy. It gives an API built upon the C language that can be used to write parallel computer programs. The GPU device operates as a coprocessor to the host i.e., CPU, running C program.

The paper is organized in six sections, in section II brief description of GPU is provided. In section III CUDA features are discussed, section IV and V provides information about fuzzy logic system and Type-2 fuzzy logic system and finally section VI provides various applications where Type-2 fuzzy logic can be used followed by conclusion.

II. GRAPHICS PROCESSING UNITS

GPU is a main hardware specially designed for highly parallel applications. The GPU's fast increase in both programmability and capability has spawned a research community that has successfully mapped a broad range of computationally demanding and complex problems to the GPU. This effort in general purpose computing with GPU, also known as GPU computing [1] [2]. GPUs have been known to users for quite a long time as a graphics rendering coprocessor to the host PC, to render cool graphic effects in multimedia based applications such as gaming, animation etc. But now the technology inside the GPU has become advanced for many computing applications other than rendering graphics. The research community

has clearly demonstrated how non-graphics-related computing can be performed on the GPUs, with more than a thousands of papers published so far in this field. So GPGPU is use of GPU computing for general purpose applications. GPU is invented by NVIDIA.

GPU computing uses GPU together with a CPU to accelerate general-purpose scientific and engineering applications. CPU sends tasks and data to GPU, GPU performs computations on data and sends back results to CPU. GPU is called as DEVICE and CPU is called as HOST. GPUs consist of thousands of smaller, more efficient cores designed for parallel performance. CPUs consist of a few cores optimized for serial processing e.g. Intel Pentium Dual Core processors have 2 cores, Quad core have 4, which are very less in number. The architecture of CPU and GPU is shown in Fig. 1. Serial portions of the code run on the CPU while parallel portions run on the GPU. GPUs contain much larger number of dedicated ALUs then CPUs. Each processing unit on GPU contains local memory that improves data manipulation and reduces fetch time.



Fig. 1 Difference Between CPU and GPU Architecture [1]

III. CUDA

CUDA is NVIDIA's solution to access the GPU. Compute Unified Device Architecture (CUDA) is a data-parallel computing environment that does not require the use of a graphics API [2]. To work on CUDA, C language is being used. A CPU and a GPU programs are build up in the same environment i.e., CUDA-C language. In CUDA multiple kernels run concurrently on a single GPU. CUDA mention each kernel as grid. A grid consists of collection of blocks. Each block runs the same kernel but independent of each other. A block contains threads, which are smallest divisible unit on a GPU. CUDA allows multiple programs, kernels; to run sequentially on a single GPU [3]. The architecture is shown in Fig. 2.

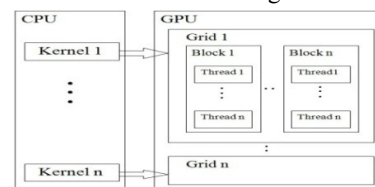


Fig. 2 CUDA Processing Model Design [1]

IV. FUZZY LOGIC SYSTEM

A Fuzzy Logic System (FLS) is able to handle the numerical data and linguistic knowledge simultaneously. FLS can be explained in form of mathematics a linear combination of fuzzy basis function and is a nonlinear universal function approximation. The fuzzy basis function expansion is very useful because its basis functions can be derived from either objective knowledge or subjective knowledge, both of which can be assigned into the forms of IF-THEN rules. Both type of knowledge can be expressed in mathematical manner. There are two types of problem knowledge which can be solved by the Fuzzy Logic System.

1. Objective Knowledge which is used for mathematical models. For example solve the formula, equations of motion for a submarine, spacecraft etc.
2. Subjective knowledge which represents linguistic information that is usually impossible to calculate value using mathematical formulas, the rules that might be valid for tracking a submarine or any other slowly moving large object etc.

FLS is a non-Linear mapping of an input data (feature) vector into a scalar output [4]. Fuzzy logic is a process that tries to simulate the “fuzzy” decision making of a person, by using fuzzy sets. Fuzzy logic often only requires a small number of fuzzy sets and a small collection of simple rules to solve the same problem. In fact, when dealing with a fuzzy problem, computers that operate using fuzzy logic often perform tasks more quickly, efficiently and in many cases better than normal computers which use traditional crisp logic. An example which illustrates the difference between fuzzy and crisp logic is the way in which a computer controls an air conditioner. The normal crisp-logic computer has a sensor that measures the temperature, after which this number is fed into a computer that has some built-in logical rules under which it operates. A fuzzy logic system has three main components:

1. Fuzzifier: A fuzzifier that takes in numbers (in this case the temperature) and transforms them into fuzzy sets.
2. Logic Control Center: A logic control center that uses rules, which are activated by fuzzy sets, and, produces fuzzy sets at its output.
3. Defuzzifier: A defuzzifier that takes the fuzzy output sets and transforms them back into numbers that indicate what action should take place, or decision should be made. Our simple fuzzy air conditioner is governed by two basic rules (real air conditioners would probably be governed by more than two rules), which use the two fuzzy input sets cold and hot. These fuzzy sets describe the temperature. The rules are associated with two fuzzy output sets, high and off which describe the settings for the air conditioner [5].

V. TYPE-2 FUZZY SYSTEMS

In this section, Type-2 fuzzy systems are presented. The structure of the Type-2 fuzzy rules is the same as for the Type-1 case because the distinction between Type-2 and Type-1 is associated with the nature of the membership functions [6]. Hence, the only difference is that now some or all the fuzzy sets involved in the rules are of Type-2. In a Type-1 fuzzy system, where the output sets are Type-1 fuzzy sets, we perform defuzzification in order to get a number, which is in some sense a crisp (type-0) representative of the combined output sets. In the Type-2 case, the output sets are of Type-2, so we have to use extended versions of Type-1 defuzzification methods [6]. The structure of Type-2 fuzzy logic system is shown below in Fig. 3.

A Type-2 membership grade can be any sub-set in [0, 1] the primary membership and corresponding to each primary membership, there is a secondary membership (which can also be in [0, 1]) that defines the possibilities for the primary membership. A Type-2 FLS is characterized by IF-THEN rules, where their antecedents or consequent sets are now of Type-2. Type-2 FLSs, can be used when the circumstances are too unknown to determine exact membership grade such as when the training data is affected by noise.

1. Fuzzifier: The fuzzifier maps a numeric vector $x = (x_1, \dots, x_p)^T \in X_1 * X_2 * \dots * X_p \equiv X$ into a Type-2 fuzzy into a Type-2 fuzzy set \tilde{A}_x in X [9], an interval Type-2 fuzzy set in this case. We use Type-2 singleton fuzzifier, in a single on fuzzification, the input fuzzy set has only a one point on non zero membership [10].
2. Rules: The structure of rules in a Type-1 FLS and a Type-2 FLS is the same, but in the latter the antecedents and the consequents is represented by Type-2 fuzzy sets [10].
3. Type reducer: The type-reducer generates a Type-1 fuzzy set output, which is then converted in a numeric output through running the defuzzifier. This Type-1 fuzzy set is also an interval set, for the case of our FLS we used center of sets (cos) type reduction [10] [8].
4. Defuzzifier: From the type- reducer, we obtain an interval set Y cos, to defuzzify it we use the average of y_l and y_r , so the defuzzified output of an interval singleton Type-2 FLS is $Y(x) = (y_{(1+y_r)})/2$.

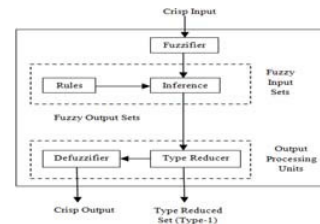


Fig. 3 Structure of a Type-2 Fuzzy Logic System [7] [8]

VI. APPLICATION OF TYPE-2 FUZZY LOGIC IN VARIOUS FIELDS

In this section a representative account of the most successful applications of Type-2 fuzzy logic in various fields is presented. Type-2 fuzzy logic has been used to allow handling higher levels of uncertainty in real world complex problems. In the applications presented in this section the superiority of Type-2 over Type-1 fuzzy logic has been shown to be significant. The applications considered in these papers are diverse, ranging from medicine to social sciences, which show the importance of the use of Type-2 fuzzy logic for this kind of problems.

In Rubio Solis, A. and Panoutsos, G. [12], an interval Type-2 radial basis function neural network (IT2-RBF-NN) is proposed as a new modeling framework. In this functional equivalence of radial basis function neural networks (RBFNN) to a class of Type-1 fuzzy logic systems (T1-FLS) to propose a new interval Type-2 equivalent system, it is systematically shown that the type equivalence (between RBF and FLS) of the new modeling structure is maintained in the case of the IT2 system. A very good computational efficiency is demonstrated as a result of the systematic and automatic creation of IT2 linguistic information and the FOU.

In the proposed approach of Melin, P., Gonzalez, C., Castro, J., Mendoza O. and Castillo O. [13], an edge detection method based on the morphological gradient technique and generalized Type-2 fuzzy logic is proposed. The theory of alpha planes is used to implement generalized Type-2 fuzzy logic for edge detection. For the defuzzification process, the heights and approximation methods are used.

In the approach of Aisbett, J. and Rickard, J.T., [14], Centroids are practically important in Type-1 and Type-2 fuzzy logic systems as a method of defuzzification and type reduction is proposed. However, computational problems arise when membership functions (MF) have singleton spikes.

In the approach of Naim, S. and Hagrass, H. [15], A general Type-2 fuzzy logic based approach for Multi-Criteria Group Decision Making, "Fuzzy Systems (FUZZ), is proposed. Multi-Criteria Group Decision Making (MCGDM) is used for viewing decision making. MCGDM is a decision tool which it is used to find a unique agreement from number of decision makers and users by evaluating the unknown judgment among them. Several fuzzy logic based approaches have been used in MCGDM to handle the linguistic uncertainties and hesitancy.

In the proposed approach of Khanesar, M.A., Kayacan, E., Kaynak, O. and Saeys, W. [16], Sliding mode Type-2 fuzzy control of robotic arm using ellipsoidal membership functions is proposed. Several papers state that the performance of the Type-2 fuzzy logic systems is superior over their Type-1 counterparts,

especially under noisy conditions. To show the effectiveness of the noise reduction capabilities of the Type-2 fuzzy logic systems, a novel Type-2 fuzzy membership function, ellipsoidal membership function, has recently been proposed.

In the proposed approach of Nguyen T., Khosravi A. Nahavandi S and Creighton D. [17], neural network and interval Type-2 fuzzy system for stock price forecasting is proposed. Stock price forecast has long been received special attention of investors and financial institutions. As stock prices are changeable over time and increasingly uncertain in modern financial markets, their forecasting becomes more important than ever before. An interval Type-2 fuzzy logic system (IT2 FLS) is employed as the second component of the hybrid forecasting method. The IT2 FLS's parameters are initialized through deployment of the k-means clustering method and they are adjusted by the genetic algorithm.

In the proposed approach of Pulido, M., Melin, P. and Castillo, O. [18], Optimization of ensemble neural networks with Type-2 fuzzy response integration for predicting the Mackey-Glass time series is proposed. The optimization of an ensemble neural network with fuzzy integration of responses based on Type-1 and Type-2 fuzzy logic is explained. Genetic algorithms are used as a method of optimization for the ensemble model in this case of study. The time series that is being considered is the Mackey-Glass benchmark.

In the proposed approach of Wati, D.A.R. and Jayanti, P.N. [19], Interval Type-2 Fuzzy Logic Controller of heat exchanger systems," Instrumentation, Communications, Information Technology, and Biomedical Engineering (ICICI-BME) is proposed. Heat exchanger systems are widely used in chemical plants. They are affected on load and disturbance of the process plant, change in operation condition and nonlinearity. In this paper, we studied the design of an interval Type-2 Fuzzy Logic Controller (FLC) for stirred tank heat exchanger systems. It controls the coolant flow in order to result in the desired temperature of output fluid.

In the proposed approach of Liang Zhao; Yanzhen Li; Yanjun Li [20], Computing with words for discrete general Type-2 fuzzy sets based on plane," Vehicular Electronics and Safety (ICVES), is proposed. General Type-2 fuzzy set (GT2FS) is the generalization of its Type-1 counterpart, which can better describe the nature of uncertainty. This paper presents computing with words (CWWs) for the discrete GT2FS based on plane representation. Firstly, it is introduced for the GT2FS. Secondly, we studied the numerical procedure of CWWs. Numerical examples are applied to assess the algorithm.

In the proposed approach of Farooq, U. and Gu, J. [21], A simple interval Type-2 fuzzy gain scheduling controller is designed for the stabilization and reference

tracking of ball and plate system. The controller employs plate angles as the premise variables for gain scheduling and its stability is guaranteed through a set of linear matrix inequalities. MATLAB simulations are performed to validate the proposed controller where it is also compared with pole placement and Type-1 fuzzy logic controllers. It is shown that the proposed controller has better response and disturbance rejection capability and is robust to measurement noise and errors.

In the proposed approach of Soto, J.; Melin, P. and Castillo, O. [22], Optimization of Interval Type-2 and Type-1 Fuzzy Integrators in Ensembles of ANFIS Models with Genetic Algorithms is proposed. An optimization of interval Type-2 and Type-1 fuzzy integrators in ensembles of ANFIS models with genetic algorithms (GAs) is represented, in this with emphasis on its application to the prediction of chaotic time series, where the goal is to reduce the prediction error. The time series that was considered is the Mackey-Glass to test the experiments.

In the proposed approach of Gonzalez, C.I [23], A new approach based on generalized Type-2 fuzzy logic for edge detection is proposed. An edge detection method based on morphological gradient technique and generalized Type-2 fuzzy logic. The theory of alpha planes is used to implement generalized Type-2 fuzzy logic. For the test we used the method of defuzzification by height and approximation. The simulation results were obtained with a Type-1 fuzzy inference system (T1FIS), an interval Type-2 fuzzy inference system (IT2FIS) and with a generalized Type-2 fuzzy logic (GT2FIS). The proposed Type-2 fuzzy edge detection method was tested with benchmark images and synthetic images.

In the proposed approach of Yunrui Bi and Srinivasan, D. [24], Single intersection signal control based on Type-2 fuzzy logic is proposed. Single intersection is generally regarded as the elementary unit for solving traffic problem. A Type-2 fuzzy logic controller (T2FLC) for single intersection signal control is presented in this paper as Type-2 fuzzy logic can handle the imprecision, uncertainties and vagueness lying in the dynamic process more efficiently.

In the proposed approach of Farooq, U [25], Design and comparison of Type-1 and interval Type-2 fuzzy gain scheduling controllers for ball and beam system is proposed. The paper presents the design and comparison of Type-1 and interval Type-2 fuzzy logic controllers for ball and beam system.

In the proposed approach of Kumbasar, T [26], A Type-2 Fuzzy Cascade Control Architecture for Mobile Robots," Systems, Man, and Cybernetics (SMC), is proposed. The real-time path tracking control of mobile robots attracted considerable research interest since they inherit non holonomic properties and uncertainties caused by the internal dynamics or feedback sensors. In

this paper, we studied a cascade control architecture, which includes the inner and outer control loops, for the path tracking control of mobile robots.

In the proposed approach of M. A. Sanchez, J. R. Castro, F. Perez-Ornelas, and O. Castillo, O [27], A hybrid method for IT2 TSK formation based on the principle of justifiable granularity and PSO for spread optimization is proposed. A new hybrid method for forming interval type 2 fuzzy inference systems (IT2 FIS) is shown. This methodology builds upon an existing Type-1 fuzzy inference system (T1 FIS) or from the output centers from any clustering algorithm, calculating the footprint of uncertainty (FOU) based on the implementation of the principle of justifiable granularity, and finally a particle swarm optimization algorithm (PSO) optimizes the spreads from First Order Takagi-Sugeno-Kang (TSK) type consequents to improve the coverage of the FOU. Focusing mainly in the coverage of the FOU, two datasets are used to demonstrate the effectiveness of FOU coverage in environments with noise, especially when the noise is on the outputs. These two datasets are a simple Fifth Order curve, and the iris benchmark dataset.

VII. CONCLUSION

In this paper a representative and concise review of Type-2 Fuzzy Logic of various applications was presented. Now's days Fuzzy Logic has got famous for using in various applications as discussed above, due to its ability to handle complexity. In this paper, a representative review of the most recent application of Type-2 Fuzzy Logic was given. Fuzzy Logic is gaining popularity due to handling of various uncertainties in the various fields. Fuzzy logic is also helpful to gain the data from the object knowledge and to predict the next result on the bases of previous knowledge.

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